Absence of activation-type conductivity in narrow quantum Hall bars

2001 J. Phys.: Condens. Matter 13 L689

(http://iopscience.iop.org/0953-8984/13/28/105)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **13** (2001) L689–L696 PII: S0953-8984(01)25915-3

LETTER TO THE EDITOR

Absence of activation-type conductivity in narrow quantum Hall bars

S Tanibayashi, S Komiyama and Y Kawaguchi

Department of Basic Science, University of Tokyo, Komaba 3-8-1, Meguroku, Tokyo 153-8902, Japan

E-mail: csusumu@asone.c.u-tokyo.ac.jp (S Komiyama)

Received 10 June 2001
Published 29 June 2001
Online at stacks.iop.org/JPhysCM/13/L689

Abstract
In narrow Hall bars, 2.3 μm wide, fabricated on a high-mobility GaAs/AlGaAs single-heterostructure crystal, the temperature dependence of the diagonal conductivity σ_{xx} in an integer quantum Hall regime ($\nu = 2$ at $B = 5.5$ T) almost vanishes below 4.2 K. This temperature-independent characteristic of σ_{xx} is interpreted as being an intrinsic property of narrow Hall bars. It is ascribed to elastic backscattering between edge states penetrating into the interior region, where the penetration depth is suggested to be the localization length, $\xi \approx 0.39$ μm, of bulk localized states. Increasing the Hall electric field E_y slowly increases σ_{xx}, which is interpreted as a consequence of E_y causing ξ to increase by promoting the hybridization of bulk localized states.

The integer quantum Hall effect (IQHE) is a remarkable phenomenon of two-dimensional electron gas (2DEG) systems in high magnetic fields, which is manifested by the vanishing of the diagonal conductivity σ_{xx} and the quantization of the Hall conductivity σ_{xy} into integer multiples of e^2/h, where e is the unit charge and h is Planck's constant [1, 2]. It is well established for conventional Hall bars with device widths larger than a few tens of micrometres that σ_{xx} strictly vanishes as the temperature, T, approaches absolute zero, but rapidly increases with increasing T due to conductivity of the thermal activation (TA) type [3] or variable-range-hopping (VRH) conductivity [4]. However, this widely known behaviour is expected to be presented only when the device width, W, exceeds the inelastic scattering length, L_{in}, because in either case the conductivity is expected to arise primarily from inelastic scattering processes.

If $W < L_{in}$, it is possible that elastic backscattering of electrons between opposite edge states [5] dominates over inelastic scattering processes in the interior 2DEG region. Since the elastic backscattering should be temperature independent in the linear transport regime, the TA- or VRH-type conductivity may be replaced by a temperature-independent conductivity. Since the backscattering may be mediated via bulk localized states [6], the localization length of bulk localized states [7], ξ, will be an important length parameter in such a coherent regime. Although deviation of the IQHE from exact quantization in narrow Hall bars has been

0953-8984/01/280689+08$30.00

reported [8], the characteristics of σ_{xx} were not studied and the role of elastic backscattering in the IQHE remains unclear.

In the case where elastic backscattering of electrons serves as a dominant mechanism for finite σ_{xx}, the effect of the Hall electric field, E_y, is of particular interest. This is because E_y may promote hybridization of localized states [9]. Experimentally, σ_{xx} is known to increase exponentially with increasing E_y [10–13]. Unfortunately, however, studying the microscopic direct effect of E_y is hampered by a possible rise in the effective electron temperature, T_e [11, 13, 14].

In this work, we study σ_{xx} in relatively narrow Hall bars. First we find that the temperature dependence of σ_{xx} practically vanishes. We estimate L_{in} from independent experiments—the results suggest that $W < L_{in}$—and interpret the findings in terms of elastic backscattering of electrons mediated by bulk localized states. Secondly, in order to exclusively study the direct microscopic effect of E_y, we study σ_{xx} by using voltage probes placed close to the electron-injecting corner of the current contact. We find a slow increase of σ_{xx} with increasing E_y, which is consistent with an E_y-induced hybridization of electron states in random potentials.

Figure 1(a) schematically shows the Hall bars studied, which are fabricated on a GaAs/AlGaAs modulation-doped single-heterostructure crystal with a sheet electron density of 2.6×10^{15} m^{-2} and the electron mobility of 80 m^2 V^{-1} s^{-1}. The main Hall-bar channel has an effective 2DEG width of $W = 2.3$ μm (3.0 μm in lithographic width) and a total length of 334 μm. The 2DEG channel is jointed at the opposite ends to 2DEG pad regions

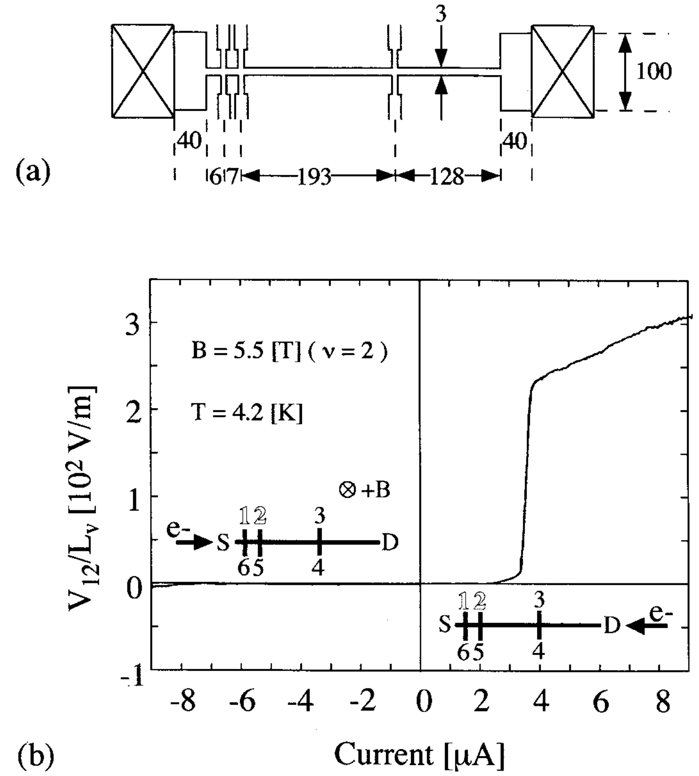

Figure 1. (a) A schematic representation of the Hall-bar sample studied. (b) The longitudinal voltage V_{12} against the current for opposite polarities in the IQHE with $\nu = 2$.

100 μm wide, from which electrons are injected or withdrawn. Voltage probes, 1 (6), 2 (5) and 3 (4), as identified in the diagrams in figure 1(b), are placed, respectively, at distances of 6, 13 and 206 μm from the left end of the Hall-bar channel. In what follows, we denote the longitudinal voltage studied with probes i and j by V_{ij}. Experiments are carried out in a T-range of 4.2 K–2.6 K at a magnetic field of $B = 5.5$ T, which corresponds to the IQHE plateau centre with the Landau level filling factor of $\nu = 2$.

A basic idea behind the present work is explained by figure 1(b), where V_{12}/L_v with the inter-probe distance $L_v = 7$ μm is displayed against the current I [15]. We note that cold electrons (at the lattice temperature T) are injected from one end of the Hall bar and travel along the channel at the drift velocity E_y/B. For 'positive polarity of current', where the injection point is the right-hand end of the 2DEG channel, the cold electrons are sufficiently heated up by E_y during the travel, reaching the region probed by contacts 1 and 2. It follows that the 2DEG system is subject to the well known 'current-induced breakdown' of the IQHE, which has been successfully explained in terms of a bootstrap-type electron heating (BSEH) [11–15]. The IQHE breakdown furnishes the curve of V_{12}/L_v versus I with a stepwise rapid increase at a critical current of about $I = 3.5$ μA. For negative polarity of I, however, the distance of travel (6–13 μm) is too short for cold electrons to be appreciably heated up, and the V_{12}/L_v versus I curve shows no signature of IQHE breakdown. It has been suggested further that the characteristic distance necessary for electron heating is typically more than 100 μm [14]. Hence, one may study σ_{xx} for cold electrons at finite E_y by investigating the values of V_{12} (V_{65}) for negative polarity of I.

We are interested not in the IQHE breakdown but in the IQHE regime in the range of small currents ($|I| < 3$ μA), where V_{12} is indiscernibly small in figure 1(b). To derive accurate values of σ_{xx} in this range, we study the derivative, $\mathrm{d}V_{12}/\mathrm{d}I$, as a function of I, by modulating I with a 0.1 μA amplitude at 100 Hz, and derive $V_{12}(I)$ through integration. Similarly, we study also V_{23}, V_{65} and V_{54} for both polarities of I. Figures 2(a) and 2(b) show, respectively for $+I$ and $-I$, values of $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2) \approx (\nu e^2/h)^2\rho_{xx}$, where $\rho_{xx} = (V_x/L_v)(W/I)$ is derived from $V_x = (V_{12} + V_{65})/2$. The current I is translated to $E_y = \rho_{xy}I/W$. (The critical current of the IQHE breakdown, $I = 3.5$ μA, corresponds to $E_y = 200$ V cm^{-1}.)

We note the following three features. First, for either polarity of I, the dependence of σ_{xx} on T is surprisingly small. In particular, σ_{xx} is practically independent of T within the experimental accuracy in the lower E_y-range below 100 V cm^{-1}. Secondly, the dependence of σ_{xx} on E_y is finite but small in the range $E_y < 100$ V cm^{-1}. Thirdly, in the higher E_y-range above 100 V cm^{-1}, σ_{xx} increases with E_y more steeply for $+I$ than for $-I$. As for the first feature, the T-independent characteristics might result from non-equilibrium population of edge states introduced by non-ohmic poor contacts [5, 16, 17]. However, this is ruled out because (i) all of the contacts have been confirmed to be nearly ideal in the present work and (ii) the data have been confirmed to remain unchanged upon reversal of the polarity of B. The third feature reveals the presence of an appreciable electron heating effect even in the lower E_y-range below the critical field $E_y = 200$ V cm^{-1} for $+I$, supporting the earlier analysis by Boisen *et al* [13]. For $-I$, however, it suggests that the 2DEG is nearly free from the heating effect.

Though not shown here, the first two features are present also if we derive σ_{xx} from $V_x = (V_{23} + V_{54})$. Hence, the absence of T-dependence as well as the weak E_y-dependence are likely to be intrinsic characteristics of narrow Hall bars. We stress that these characteristics are in marked contrast to the earlier-reported TA-type conductivity empirically described as [11, 12]

$$\sigma_{xx}(T, E_y) \approx A\exp[-(\hbar\omega_c - eXE_y)/2kT] \qquad (1)$$

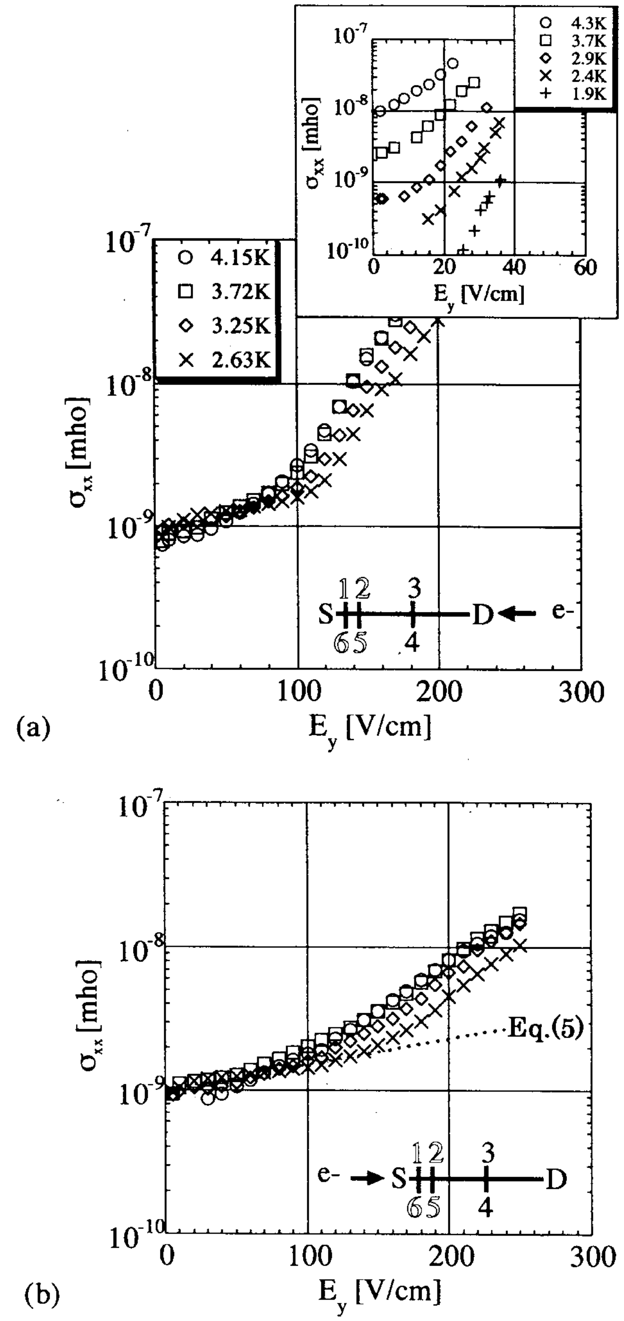

Figure 2. σ_{xx} versus E_y at different temperatures, for positive polarity (a) and negative polarity (b) of the current. The inset shows similar data taken for a wider Hall bar [11].

where $A = e^2/h$ [3]. The inset of figure 2(a) displays an example for a Hall bar 40 μm wide at $B = 3.6$ T ($\nu = 4$) [11]. Here, $X \approx 0.7$ μm ($B = 3.6$ T) is a constant and ($\hbar\omega_c = eB/m^*$ is the Landau level energy spacing with B the magnetic field and m^* the effective electron mass. The TA conductivity described by equation (1) would predict a decrease of σ_{xx} by a factor about 2×10^3 as T decreases from 4.15 K to 2.63 K for $\hbar\omega_c = 9.4$ meV ($B = 5.5$ T) and $E_y = 0$. In the case of the VRH conductivity as well, the expected reduction is by more than one order in magnitude. The data of figure 2 make it probable that the earlier-reported E_y-dependence (equation (1)) arose primarily from the heating effect of electrons [13].

In the following discussion, we will concentrate our attention on the data taken with negative polarity of the current in the range $E_y < 100$ V cm^{-1} (figure 2(b)), where we suppose the phenomenon to be substantially free from the effect of T_e.

We first suggest that our conductor is in a coherent regime in the limit of small E_y. The absolute size of the inelastic scattering length, L_{in}, of electrons in a bulk Landau level has been derived by Machida *et al* through experimental studies of device-size scaling in IQHE transition regions [18]. The values derived are expressed as $L_{in} = DT^{-1.5}$ with $D = 1.0$ μm K$^{1.5}$ for a high-mobility 2DEG system at $B = 2.4$ T. We expect L_{in} to be, roughly, inversely proportional to the density of states (DOS) of the Landau levels. It follows that L_{in} will take a maximum value in the IQHE regime where the DOS takes a minimum value, as schematically depicted with a solid line in figure 3(a). Scaling the result of Machida *et al* by assuming a Gaussian DOS:

$$\mathrm{DOS}(\varepsilon) = (\pi\ell_B^2)^{-1}\left[(1-x)\sum_n (2\pi)^{-1/2}\Gamma^{-1}\exp-\{(\varepsilon-\varepsilon_n)/\sqrt{2}\Gamma\}^2 + x/\hbar\omega_c\right] \tag{2}$$

with a constant background of $x = 0.05$–0.1 and $\Gamma/(\hbar\omega_c) = 0.05$–$0.1$, we estimate $L_{in} \approx 5$–10 μm under the present experimental conditions at $T = 4.2$ K ($B = 5.5$ T). Since $L_{in}(T, \varepsilon)$ rapidly increases as T decreases, the estimation above shows that the inequality $L_{in} > W$ holds in the range of experimental temperature, explaining the absence of T-dependence for σ_{xx}.

Let us consider the mechanism of T-independent σ_{xx}. We note that σ_{xx} takes relatively large values, $\sigma_{xx} \approx 1 \times 10^{-9}$ Ω^{-1}. In coherent conductors, σ_{xx} would strictly vanish if elastic backscattering of electrons in one edge state to the opposite edge state is absent [5]. In true physical conductors with random potentials, edge states are expected to penetrate into the interior 2DEG region by being hybridized with localized bulk states. A suitable measure of penetration is the localization length, $\xi(\varepsilon_F)$, of bulk localized states [6], and an average profile of the envelope function of the wave function in the widthwise y-directions may be approximated as $|\Psi|^2 = \exp(-2y/\xi)$. Hence, we expect elastic backscattering to set in when the device width, W, is reduced to a size comparable to $\xi(\varepsilon_F)$. To estimate ξ, we note that ξ diverges at the centre of each Landau level, $\xi(\varepsilon) \propto |\varepsilon - \varepsilon_c|^{-\nu}$ [7], as schematically represented by a dotted line in figure 3(a). Here, the critical exponent ν is experimentally reported to be $\nu = 2$–2.3 [6, 19] in the vicinity of ε_c. We can estimate $\xi(\varepsilon_F)$ under the present experimental conditions, $\varepsilon_F = \varepsilon_c \pm \hbar\omega_c/2$, by extrapolating the data of Machida and Komiyama for IQHE transition regions ($\varepsilon_F \approx \varepsilon_c$) [6] to $\varepsilon_F = \varepsilon_c \pm \hbar\omega_c/2$. Assuming that the relation $\xi/\ell_B \propto |\varepsilon - \varepsilon_c|^{-\nu}$ holds outside transition regions and applying the DOS described by equation (2), we obtain $\xi(\varepsilon_F) = 0.2$–0.4 μm; these values are much larger than the magnetic length $\ell_B = (\hbar/e|B|)^{1/2} = 0.011$ μm at $B = 5.5$ T, and not negligibly small compared to W.

Let us examine whether the estimated values of ξ given above are consistent with the observed amplitude of σ_{xx}. If a conductor segment of length L and width W is connected to ideal leads at the opposite ends, the probability of reflection of electrons entering the conductor segment will be given by

$$1 - T = (L/\xi)\exp(-2W/\xi) \tag{3}$$

which gives

$$\sigma_{xx} = (2e^2/h)(W/\xi)\exp(-2W/\xi) \tag{4}$$

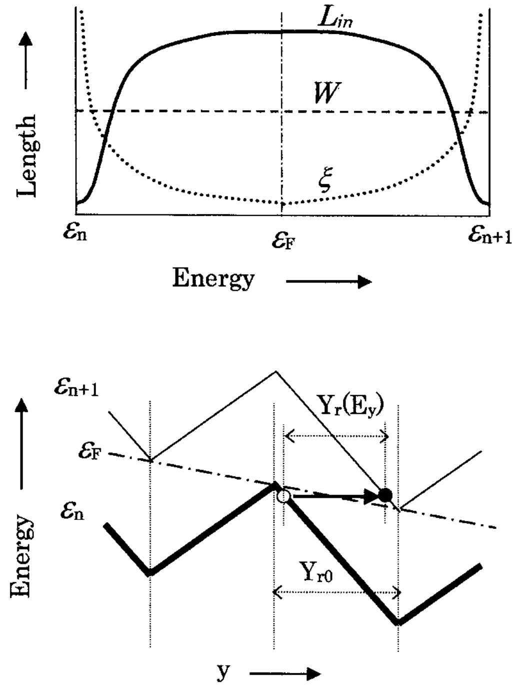

Figure 3. (a) A schematic representation of the inelastic scattering length, L_{in}, the localization length, ξ, and the device width, W, as functions of energy, where ε_n and ε_{n+1} denote Landau level centres. (b) A simplified Landau level energy profile in long-range random potentials.

through the relation $T = [1 + (\rho_{xx}/\rho_{xy})(L/W)]^{-1}$ [20, 21]. Equation (4) predicts $\sigma_{xx} = 1 \times 10^{-9}\ \Omega^{-1}$ ($T = 1.3 \times 10^{-5}$) if $\xi = 0.36\ \mu$m is assumed for $W = 2.3\ \mu$m. Hence, the estimated values, $\xi = 0.2$–$0.4\ \mu$m, are consistent with the experimentally found amplitude of σ_{xx}, leading us to suggest that σ_{xx} is caused by the elastic backscattering of electrons between opposite edge states.

We now wish to interpret the slow increase of σ_{xx} with increasing E_y. The dependence found experimentally is empirically described by

$$\sigma_{xx}(E_y)/\sigma_{xx}(0) = \exp(eYE_y/\hbar\omega c) \qquad (5)$$

as shown by a dotted straight line in figure 2(b), where we take $Y = 0.49\ \mu$m with $\hbar\omega_c = 9.3$ meV ($B = 5.5$ T) to fit the experimental data. Two possible mechanisms may be considered, both of which are related to Zener-type elastic tunnelling of electrons [22, 23]. First, Zener-type tunnelling will promote inelastic scattering processes through creation of non-equilibrium electrons. If $L_{in} < W$, σ_{xx} will thereby increase. Secondly, the process of elastic tunnelling between different Landau levels will be promoted by E_y leading to (coherent) hybridization of localized states. This works to increase ξ with increasing E_y in a coherent regime ($W < L_{in}$).

In any case, however, the Zener-type tunnelling is unrealistic if random potentials are ignored: without random potentials, $Y \approx \ell_B = (\hbar/e|B|)^{1/2} = 0.011\ \mu$m is predicted by equation (5), which has never been observed in existing experiments, and is of course far from the present finding, $Y \approx 45\ell_B$.

We consider below a simplified picture that takes account of the effect of random potentials. Slowly varying random potentials produce compressible and incompressible 2DEG subregions of submicron scales in the IQHE regime [24], as has been visualized via micro-probing techniques [25–27]. The existence of such structures implies that the Landau level energy fluctuates spatially with an amplitude of $\hbar\omega_c$ and that the highest occupied Landau level and the lowest unoccupied Landau level touch the Fermi level forming a framework for the structure. Figure 3(b) schematically illustrates the energy diagram, where we let Y_{r0} be the characteristic distance between the top and an adjacent bottom in the fluctuation potential. The local Fermi level has a slope $\varepsilon_F = -eyE_y$ in the widthwise y-direction. The distance, $Y_r(E_y)$, over which an electron jumps in a Zener-type elastic tunnelling process decreases with increasing E_y as $Y_r(E_y) = Y_{r0}/[1 + (eY_{r0}E_y/\hbar\omega_c)]$. Noting the overlap integral between the initial- and final-state wave functions, we expect the probability of such an elastic tunnelling process to be, roughly, proportional to $\exp[-(1/2)Y_r(E_y)/\ell_B{}^2]$, which leads to equation (5) with

$$Y = (Y_{r0}/\ell_B)^2 Y_{r0} \tag{6}$$

on noting that $\ell_B \ll Y_{r0}$ or $eY_{r0}E_y \ll \hbar\omega_c$. Taking the experimental value, $Y = 0.49$ μm, we have $Y_{r0} = 0.039$ μm from equation (6), which is comparable to the value estimated from independent experiments [14]. The value also appears to be consistent with a size of Y_{r0} conjectured from direct images of similar 2DEG systems [25–27].

The above discussion suggests that a Zener-type elastic tunnelling process in slowly varying random potentials is the origin of the slow increase of σ_{xx} with increasing E_y found experimentally. Among the two specific mechanisms discussed in the paragraph including equation (5), the second mechanism, in which ξ increases with E_y while the conductor remains coherent ($W < L_{in}$), is suggested to be more probable, because of the absence of any substantial T-dependence in the finite E_y-range (<100 V cm^{-1}).

Finally, let us estimate the general critical width of the Hall bar, below which the T-independent elastic backscattering dominates over the TA conductivity. Comparing the σ_{xx}s given in equations (1) and (4) and ignoring a small logarithmic term, we can predict the elastic-backscattering-induced conductivity to dominate over the TA conductivity when W is smaller than

$$W_c = (1/4)(\hbar\omega_c/kT)\xi. \tag{7}$$

Although values of ξ in different conditions are not very clear, the enhancement factor in equation (7), $\hbar\omega_c/kT$, readily exceeds 10^2 in familiar experimental conditions. If we assume ξ to be of the order of a few submicrons (like in the present work), W_c can be larger than 10 μm if $T < 1$ K and $B > 5$ T. Thus the elastic backscattering between edge states is possibly a general mechanism determining σ_{xx} at low T.

In summary, we have found that σ_{xx} in IQHE Hall bars 2.3 μm wide ($\nu = 2$, $B = 5.5$ T) does not show substantial T-dependence below 4.2 K, and that it slowly increases with increasing E_y. The amplitude of σ_{xx} (1×10^{-9} Ω^{-1}) in the limit of low E_y has been reasonably explained in terms of the elastic backscattering of electrons between edge states that are broadened widthwise through hybridization with bulk localized states ($\xi = 0.36$ μm). The increase of σ_{xx} with E_y has been ascribed to E_y promoting the hybridization of bulk localized states in random potentials.

References

[1] von Klitzing K, Dorda G and Pepper M 1980 *Phys. Rev. Lett.* **45** 449

[2] Kawaji S and Wakabayashi J 1981 *Physics in High Magnetic Fields* ed S Chikazumi and N Miura (Berlin: Springer) p 284

[3] Clark R G, Mallett J R and Haynes S R 1988 *Phys. Rev. Lett.* **60** 1747

[4] Briggs A, Guldner Y, Vieren J P, Voos M, Hirtz J P and Razeghi M 1983 *Phys. Rev.* B **27** 6549

[5] Büttiker M 1988 *Phys. Rev.* B **38** 9375

[6] Machida T and Komiyama S 2000 *Phys. Rev.* B **62** 13 004

[7] Aoki H and Ando T 1986 *Surf. Sci.* **170** 249
Pruiskin A M M 1988 *Phys. Rev. Lett.* **61** 1297
Huckestein B and Kramer B 1990 *Phys. Rev. Lett.* **64** 1437

[8] Timp G *et al* 1987 *Phys. Rev. Lett.* **59** 732
Chang A M *et al* 1988 *Solid State Commun.* **67** 769

[9] Trugman A 1983 *Phys. Rev.* B **27** 7539

[10] Ebert G, von Klitzing K, Ploog K and Weimann G 1983 *J. Phys. C: Solid State Phys.* **16** 5441

[11] Komiyama S, Takamasu T, Hiyamizu S and Sasa S 1985 *Solid State Commun.* **54** 479

[12] Takamasu T, Komiyama S, Hiyamizu S and Sasa S 1986 *Surf. Sci.* **170** 202

[13] Boisen A, Boggild P, Kristensen A and Lindelof D P E 1994 *Phys. Rev.* B **50** 1957

[14] Komiyama S and Kawaguchi Y 2000 *Phys. Rev.* B **61** 2014

[15] Komiyama S, Kawaguchi Y, Osada T and Shiraki Y 1996 *Phys. Rev. Lett.* **77** 558

[16] Komiyama S and Hirai H 1989 *Phys. Rev.* B **40** 7767

[17] Komiyama S, Hirai H, Sasa S and Fujii T 1990 *Solid State Commun.* **73** 91

[18] Machida T, Hirai H, Komiyama S and Shiraki Y 1996 *Phys. Rev.* B **54** 16 860

[19] Koch S, Haug R J, von Klitzing K and Ploog K 1991 *Phys. Rev. Lett.* **67** 883

[20] McEuen P L, Szafer A, Richter C A, Alphenaar B W, Jain J K, Stone A D and Wheeler R G 1990 *Phys. Rev. Lett.* **64** 2062

[21] Komiyama S and Nii H 1993 *Physica* B **184** 7

[22] Tsui D C, Dolan G J and Gossard A C 1983 *Bull. Am. Phys. Soc.* **28** 365

[23] Heinonen O, Taylor P L and Girvin S M 1984 *Phys. Rev.* B **30** 3016

[24] Chklovski D B and Lee P A 1993 *Phys. Rev.* B **48** 18 060

[25] Tessmer S H, Glicofridis P I, Ashoori R C, Levitov L S and Melloch M R 1998 *Nature* **392** 51

[26] Finkelstein G, Glicofridis P I, Ashoori R C and Shayegan M 2000 *Science* **289** 90

[27] Zhitenev N B, Fulton T A, Yacoby A, Hess H F, Pfeiffer L N and West K W 2000 *Nature* **404** 473